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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

364. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The English physicist, Hooke, published the discovery contained in the Latin sentence, "Ut tensio sic vis" by the cypher *ceiiinosssttuv*. Preserving the lexicographical order, find which permutation, taking all letters, the Latin sentence is from the cypher.

Solution by H. C. FEEMSTER, A. M., York College, York, Nebraska.

Beginning at the first, and counting all permutations, although letters may be alike, the permutation required is:

$$12\underline{|13}+10\underline{|12}+\underline{|10}+4\underline{|9}+5\underline{|8}+\underline{|7}+3\underline{|6}+3\underline{|5}+\underline{|4}+2\underline{|2}+\underline{|1}$$
$$=79,\ 519,\ 555,\ 109.$$

370. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Prove that, if the fraction m/p (p prime) gives a recurring decimal with an even number of digits in the cycle, the sum of the two halves will be composed of 9's. (The special case where the number$=p-1$ was proposed in the MONTHLY, Vol. IV, and answered in Vol. V, p. 11. The proof there given, however, is not complete.) The above property is true of other fractions, *e. g.*, $\frac{1}{77}=.\dot{0}1298\dot{7}$, $\frac{1}{133}=\frac{1}{7.19}=.\dot{0}07518796,99248120\dot{3}$. Find for what fractions this is true.

I. Solution by F. H. SAFFORD, Ph. D., University of Pennsylvania.

1°. To prove that the sum of the two halves will be composed of 9's. Let the number of digits in the cycle be $2n$, then the number of 9's in the denominator of the unreduced common fraction will be $2n$, and this sequence is divisible by the sequence of n 9's, *e. g.*, 999999 is divisible by 999. It follows that the unreduced numerator must be also divisible by the sequence of n 9's. Suppose the original cycle is $\dot{a}bcde\dot{f}$, the fraction may be written

$$\frac{(abc)(1000)+(def)}{999999},\ \text{or}\ \frac{(abc)(999)+(abc)+(def)}{999999},$$

in which (abc), etc., represents a number of three digits and not a product. The part $(abc)+(def)$ must be divisible by 999 as shown by the preceding.

Not all of the digits a, b, c, d, e, f, can be 9's, so that the sum is less than twice 999 and is thus 999. Hence for a given (abc) the corresponding (def) is fixed and has the specified property. The proof is extensible to any case and does not require that the recurring decimal be perfect.

2°. The results of the indicated cancellations are:

$$\frac{a+1}{11},\ \frac{(ab)+1}{101},\ \frac{(abc)+1}{1001},\ \text{etc.}$$

Of such fractions, those denominators (except the first) having an even number of digits are composite, also others such as $10001=73.137$. The numerators $(abc...)+1$ are unrestricted as might be anticipated, so that fractions having denominators of the form 10^n+1 have the required property. The illustration $\frac{1}{77}=.\dot{0}1298\dot{7}$ came from $\frac{012+1}{1001}$; the fraction $\frac{1}{133}$ originally had the numerator $007518796+1$ and the denominator 10^9+1.

II. Solution by the PROPOSER.

Let the number of digits in the cycle be $2n$. Let d be the value of the recurring decimal.

$d=.abc...(n \text{ digits})...\alpha\beta\gamma...(n \text{ digits})...$

Then $10^{2n}d=abc...\alpha\beta\gamma...\ .abc...\alpha\beta\gamma...$; $d=.abc...\alpha\beta\gamma...$

Subtracting, we have, $(10^{2n}-1)d=abc...\alpha\beta\gamma...$ an integer.

$\therefore d=\frac{abc...\alpha\beta\gamma...}{10^{2n}-1}=\frac{m}{p}$ by hypothesis.

Then $\frac{m(10^{2n}-1)}{p}=\frac{m(10^n-1)(10^n+1)}{p}$ is an integer $=abc...\alpha\beta\gamma...$

p cannot divide 10^n-1 since in that case the period of m/p would be n and not $2n$. Therefore p is a factor of 10^n+1.

$$\begin{array}{rl} 10^n(m/p)= & abc...\ .\alpha\beta\gamma...abc... \\ (m/p)= & \quad .abc...\alpha\beta\gamma... \\ \hline (10^n+1)(m/p)= & abc...\ =\text{an integer by proof just given.} \end{array}$$

Therefore, the sum of the two decimals $.\alpha\beta\gamma...$ and $.abc...$ must be $.999...$

The answer to the second part of the question follows at once from the above. It is sufficient if p instead of being a prime is *any* factor of 10^n+1. $10^3+1=7.11.13$. Therefore if $p=77$, 91, 143, or 1001, a/p will have six figures in the cycle and the two halves will be complements.

III. Solution by C. A. LAISANT, Paris, France.

If the fraction m/pp' gives a periodical decimal fraction with an even number $2n$ of digits, and if we write the period $A.10^n+B$, we have

$$\frac{m}{pp'}=\frac{A.10^n+B}{(10^n-1)(10^n+1)}.$$

10^n-1 cannot be mult. p and mult. p', because the period would then have n digits, instead of $2n$.

Suppose 10^n+1 mult. p and mult. p', hence mult. pp' (supposing p, p' primes together). Writing $10^n+2=qpp'$, we have

$$mq=\frac{A.10^n+B}{10^n-1}=A+\frac{A+B}{10^n-1}.$$

But $A<10^n$, $B<10^n$. Then, $A+B=10^n-1$, and the property is true.

If $10^n+1=$mult. p, and if p', divisor of 10^n-1, is 3 or 9, the sum $A+B$ is a number with similar digits.

Ex. $\frac{1}{21}=\frac{1}{7.3}=0{,}047619...$ $\qquad 047+619=666.$

$\frac{1}{63}=\frac{1}{7.9}=0{,}015373...$ $\qquad 015+373=888.$

Also solved by J. Scheffer.

H. C. Feemster should have received credit for solving 368.

GEOMETRY.

390. Proposed by PROF. R. C. ARCHIBALD, Brown University, Providence, R. I.

Find, geometrically, and without introducing focal properties, the locus of the vertices of the conjugate parallelograms of an ellipse.

Solution by the PROPOSER.

The solution which I had in mind when I proposed this problem was the following: By the elementary ideas of a *stretch* in projective geometry, if the radius of a circle, α, be b, and the stretch ratio be $\frac{a}{b}$ ($a>b$, say), α will be transformed into an ellipse with semi-axes of lengths a and b. Corresponding to any conjugate parallelogram of the ellipse will be a square circumscribing α; and since the locus of the vertices of all such squares is another circle concentric with α and of radius $\sqrt{(2)}b$, the stretch which transforms α into the ellipse with axes a and b will transform the concentric circle with radius $\sqrt{(2)}b$ into the ellipse with axes $\sqrt{(2)}a$, $\sqrt{(2)}b$. And this is the required locus, as found *analytically* by Mr. Scheffer (AMERICAN MATHEMATICAL MONTHLY, Vol. XIX, p. 54, March, 1912).